

# ACT Math test – Trigonometry Review

Many students are a little scared of trig, but the ACT seems to overcompensate for that fact by testing trig in an extremely straightforward way. ACT trig is basically all about right triangles. When it comes down to it, you only have to be comfortable with the most basic aspects of trig to do well on the ACT trig questions.

Finally, there will only be four trig questions on the Math Test, so even if you aren't comfortable with trig, it won't destroy your Math score. The topics of trigonometry covered by the ACT are:

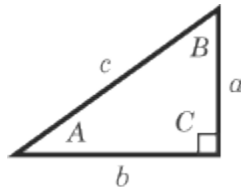
1. SOHCAHTOA
2. Solving Triangles
3. Trigonometric Identities
4. Trigonometric Graphs

## SOHCAHTOA: Sine, Cosine, and Tangent

If you can remember the acronym SOHCAHTOA, you'll do really well on the trig questions. Yup, it's as easy as that. This acronym captures almost everything you'll need to know to answer ACT trig questions. It means:

SOH:	Sine	(Opposite over Hypotenuse)
CAH:	Cosine	(Adjacent over Hypotenuse)
TOA:	Tangent	(Opposite over Adjacent)

All of this opposite-adjacent-hypotenuse business in the parentheses tells you how to calculate the sine, cosine, and tangent of a right triangle. Opposite means the side facing the angle; adjacent means the side that's next to the angle, but isn't the hypotenuse (the side opposite the  $90^\circ$  angle). Say you have the following right triangle:



If you want to find the sine of  $A$  just think of SOH, and you know you have to divide  $a$ , the opposite side, by  $c$ , the hypotenuse of the triangle. Get the idea? So in the above:

$$\begin{aligned}\sin A &= \frac{a}{c} & \sin B &= \frac{b}{c} \\ \cos A &= \frac{b}{c} & \cos B &= \frac{a}{c} \\ \tan A &= \frac{a}{b} & \tan B &= \frac{b}{a}\end{aligned}$$

There are some values for the sine, cosine, and tangent of particular angles that you should memorize for the ACT. ACT trig questions often test these angles, and if you have the trig values memorized, you can save a great deal of time.

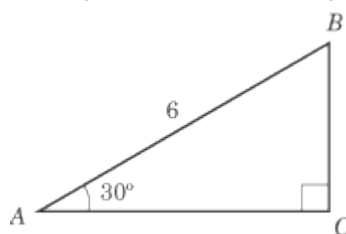
## ACT Math test – Trigonometry Review

Angle	Sine	Cosine	Tangent
0°	0	1	0
30°	1/2	$\sqrt{3}/2$	$\sqrt{3}/3$
45°	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60°	$\sqrt{3}/2$	1/2	$\sqrt{3}$
90°	1	0	undefined

### Solving Triangles

Once you understand the trigonometric functions of sine, cosine, and tangent, you should be able to use these functions to “solve” a triangle. In other words, if you are given some information about a triangle, you should be able to use the trigonometric functions to figure out the values of other angles or sides of the triangle. For example,

What is the length of  $\overline{BC}$  in the triangle below?



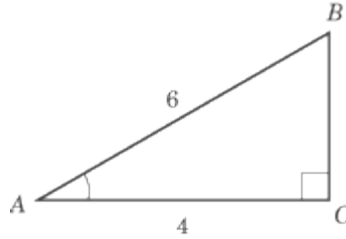
In this problem, you are given the measure of  $\angle A$ , as well as the length of  $\overline{AB}$ . The image also shows that this triangle is a right triangle. You can use this information to solve for  $\overline{BC}$  if you can figure out which trigonometric function to use. You have to find the value of side  $\overline{BC}$ , which stands opposite the angle you know. You also know the value of the hypotenuse. To figure out  $\overline{BC}$ , then, you need to use the trig function that uses both opposite and hypotenuse, which is sine. From the chart of the values of critical points, you know that  $\sin 30^\circ = 1/2$ .

To solve:

$$\begin{aligned}\sin 30^\circ &= \frac{x}{6} \\ \frac{1}{2} &= \frac{x}{6} \\ x &= 3\end{aligned}$$

Another favorite ACT problem is to combine the Pythagorean theorem with trig functions, like so:

What is the sine of  $\angle A$  in right triangle  $ABC$  below?



To find the sine of  $\angle A$ , you need to know the value of the side opposite  $\angle A$  and the value of the hypotenuse. The figure gives the value of the hypotenuse, but not of the opposite side. However, since the figure *does* provide the value of  $\overline{AC}$ , you can calculate the value of the opposite side,  $\overline{BC}$ , by using the Pythagorean theorem.

$$\begin{aligned}AB^2 &= AC^2 + BC^2 \\6^2 &= 4^2 + x^2 \\36 &= 16 + x^2 \\x^2 &= 20 \\x &= \sqrt{20} = 2\sqrt{5}\end{aligned}$$

Now that you know the value of  $\overline{BC}$ , you can solve for sine  $A$ :

$$\begin{aligned}\sin A &= \frac{2\sqrt{5}}{6} \\ \sin A &= \frac{\sqrt{5}}{3}\end{aligned}$$

## Trigonometric Identities

A trigonometric identity is an equation involving trigonometric functions that holds true for all angles. For the ACT test, trigonometric identities, on those few occasions when they come up, will be helpful in situations when you need to simplify a trigonometric expression. The two identities you should know are:

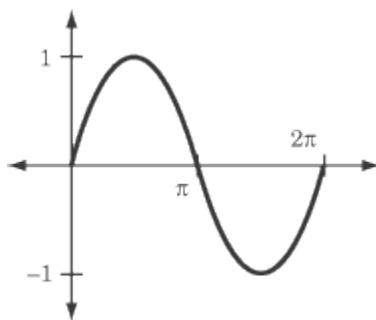
1.  $\tan \theta = \frac{\sin \theta}{\cos \theta}$
2.  $\sin^2 \theta + \cos^2 \theta = 1$

If you see an expression that contains either  $(\sin \theta)/(\cos \theta)$  or  $\sin^2 \theta + \cos^2 \theta$ , you should immediately substitute in its identity.

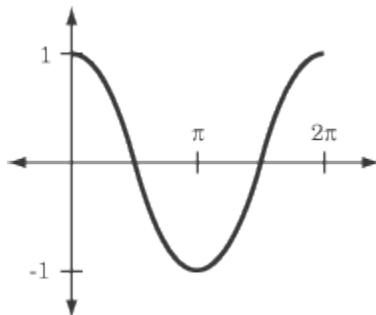
## Trigonometric Graphs

The ACT will include one or two questions covering the graphs of the trigonometric functions. You should be able to match each graph with each function, and you should know when the different functions reach their highest point and lowest point.

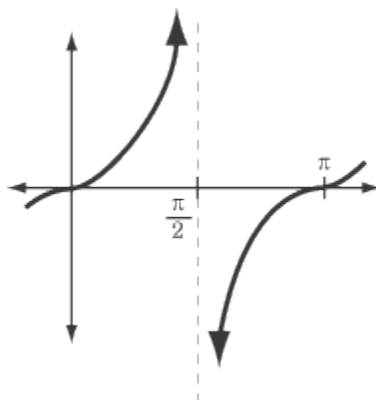
## Graph of $y = \sin x$



## Graph of $y = \cos x$



## Graph of $y = \tan x$



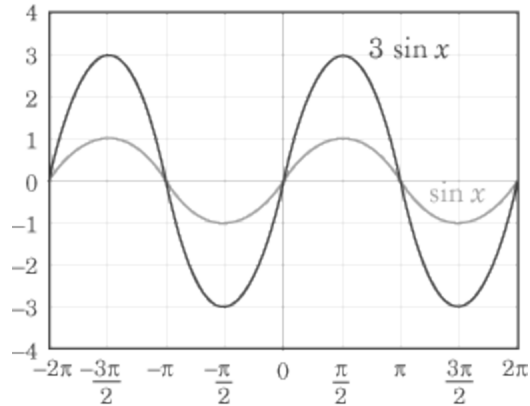
## Stretching the Trigonometric Graphs

In addition to knowing the graphs of the trigonometric functions, you should also know how the graphs can be stretched vertically or horizontally. Vertical stretches affect the graph's amplitude, while horizontal stretches change the period.

### STRETCHING THE AMPLITUDE

If a coefficient is placed in front of the function, the graph will stretch vertically: its highest points will be higher and its lowest points will be lower. Whereas the function  $y = \sin x$  never goes higher than 1 or lower than -1, the function  $y = 3 \sin x$  has a high point of 3 and a low point of -3. Changing the amplitude of a function does not change the value of  $x$  at which the high and low points occur. In the figure below, for example,  $y = \sin x$  and  $y = 3 \sin x$  both have their high points when  $x$  equals  $-3\pi/2$  and  $\pi/2$ .

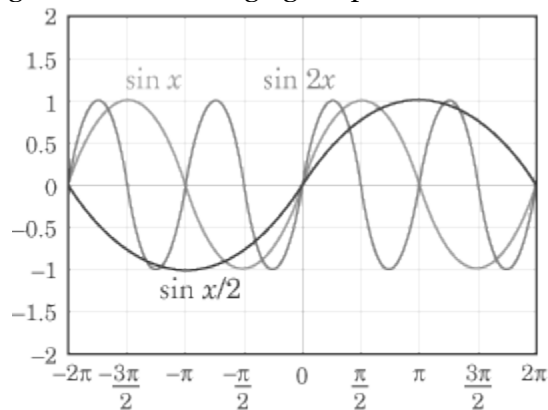
## ACT Math test – Trigonometry Review



The amplitude of a trigonometric function is equal to the absolute value of the coefficient that appears before the function. The amplitude of  $y = 2 \cos x$  is 2, the amplitude of  $y = \frac{1}{2} \sin x$  is  $\frac{1}{2}$ , and the amplitude of  $y = -2 \sin x$  is 2.

### STRETCHING THE PERIOD

If a coefficient is placed before the  $x$  in a trigonometric function, the function is stretched horizontally: its curves become steeper or less steep depending on the coefficient. The curves of  $y = \sin 3x$  are steeper than the curves of  $y = \sin(\frac{1}{2}x)$ . This coefficient doesn't affect the amplitude of the function in any way, but it does affect *where* on the  $x$ -axis the function has its high and low points. The figure on the next page shows how changing the period affects a sine curve.



The ACT may test your knowledge of periods by presenting you with a trig function that has a period coefficient and asking you for the smallest positive value where the function reaches its maximum value. For example:

What is the smallest positive value for  $x$  where  $y = \cos 2x$  reaches its maximum value?

To answer this question, you need to know the original cosine curve and be able to carry out some very easy math. Knowing the original trig graph is the crucial thing; the math, as we said, is easy.